

**Math 144 – Finite Mathematics**  
**FINAL EXAM REVIEW**

**Problem 1.** Perform the following row operation on the indicated matrix.

(a)  $\left[ \begin{array}{ccc|c} -1 & 3 & 2 & 4 \\ 4 & 0 & 1 & -3 \\ 2 & -4 & 8 & -6 \end{array} \right]; R_1 + \frac{1}{2}R_3 \rightarrow R_3$

(b)  $\left[ \begin{array}{ccc|c} 3 & -2 & 11 & -7 \\ -3 & 5 & -8 & 21 \\ 8 & -11 & 2 & 9 \end{array} \right]; R_1 + R_2 \rightarrow R_2$

**Problem 2.** Solve the following systems of equations using Gauss-Jordan elimination.

(a)  $\begin{cases} x - y + 3z = 13 \\ 4x + y + 2z = 17 \\ 3x + 2y + 2z = 1 \end{cases}$

(b)  $\begin{cases} x + 2y + 3z = 9 \\ x - 2y = 4 \\ 3x + 2z = 12 \end{cases}$

(c)  $\begin{cases} x - 2z = 5 \\ 3x + 2y = 8 \\ -x + 2z = 10 \end{cases}$

**Problem 3.** At the beginning of a laboratory experiment, five baby rats measured 5.6, 6.4, 6.9, 7.6, and 6.1 cm in length, and weighed 144, 138, 149, 152, and 146 g, respectively.

- (a) Write a  $2 \times 5$  matrix using this information.
- (b) At the end of two weeks, their lengths (in centimeters) were 10.2, 11.4, 11.4, 12.7, and 10.8, and their weights (in grams) were 196, 196, 225, 250, and 230. Write another  $2 \times 5$  matrix with this information.
- (c) Use the matrices in parts (a) and (b) to write a matrix that gives the amount of change in length and weight for each rat.

**Problem 4.** Let  $A, B, C, D, E,$  and  $F$  be the following matrices with real entries:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix},$$
$$D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}.$$

Compute each of the following, if it exists. If it doesn't exist, explain why.

- (a)  $AB$
- (b)  $EF$
- (c)  $CB + D$
- (d)  $AB + D^2$ , where  $D^2 = DD$ .

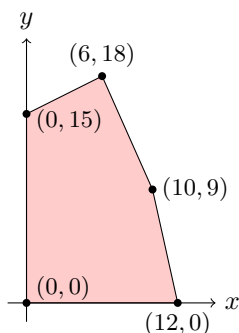
**Problem 5.** Graph the feasible region for each system of inequalities.

(a)  $\begin{cases} x + y \leq 7 \\ x - y \leq -4 \\ 4x + y \geq 0 \end{cases}$

(b)  $\begin{cases} -2 < x < 3 \\ -1 \leq y \leq 5 \\ 2x + y < 6 \end{cases}$

(c)  $\begin{cases} 0 \leq x \leq 9 \\ x - 2y \geq 4 \\ 3x + 5y \leq 30 \\ y \geq 0 \end{cases}$

**Problem 6.** The following graph shows a region of feasible solutions. Determine the maximum and minimum values of each objective function over the feasible region.



(a)  $z = 0.35x + 1.25y$

(b)  $z = 1.5x + 0.5y$

(c)  $z = 4x + 2y$

(d)  $z = x + 6y$

**Problem 7.** Solve the following linear programming problem using any method.

A certain predator requires at least 10 units of protein and 8 units of fat per day. One prey of species I provides 5 units of protein and 2 units of fat; one prey of species II provides 3 units of protein and 4 units of fat. Capturing and digesting each species-II prey requires 3 units of energy, and capturing and digesting each species-I prey requires 2 units of energy. How many of each prey would meet the predator's daily food requirements with the least expenditure of energy?

**Problem 8.** Set up, but do not solve, an initial simplex tableau for each problem.

- (a) A baker has 150 units of flour, 90 of sugar, and 150 of raisins. A loaf of raisin bread requires 1 unit of flour, 1 of sugar, and 2 of raisins, while a raisin cake needs 5, 2, and 1 units, respectively. Raisin bread sells for \$1.75 a loaf and raisin cake for \$4.00 each.
- (b) The Cut-Right Company sells sets of kitchen knives. The Basic Set consists of 2 utility knives and 1 chef's knife. The Regular Set consists of 2 utility knives and 1 chef's knife, and 1 slicer. The Deluxe Set consists of 3 utility knives, 1 chef's knife, and 1 slicer. Their profit is \$30 on a Basic Set, \$40 on a Regular Set, and \$60 on a Deluxe Set. The factory has on hand 800 utility knives, 400 chef's knives, and 200 slicers.

**Problem 9.** The initial simplex tableau for a linear programming problem is provided. Use the simplex method to solve each problem.

(a) 
$$\left[ \begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & \\ \hline 3 & 3 & 2 & 1 & 0 & 0 & 18 \\ 2 & 2 & 3 & 0 & 1 & 0 & 16 \\ \hline -4 & -6 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

(b) 
$$\left[ \begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\ \hline 2 & 2 & 8 & 1 & 0 & 0 & 0 & 40 \\ 4 & -5 & 6 & 0 & 1 & 0 & 0 & 60 \\ 2 & -2 & 6 & 0 & 0 & 1 & 0 & 24 \\ \hline -14 & -10 & -12 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

**Problem 10.** State the dual to each LP problem. Do not solve the dual problem.

- |                      |                                      |                      |                                      |
|----------------------|--------------------------------------|----------------------|--------------------------------------|
| (a) <b>Maximize:</b> | $z = 10x_1 + 12x_2$                  | (b) <b>Minimize:</b> | $w = 2y_1 + y_2 + 3y_3$              |
| <b>subject to:</b>   | $4x_1 + 2x_2 \leq 20$                | <b>subject to:</b>   | $y_1 + y_2 + y_3 \geq 100$           |
|                      | $5x_1 + x_2 \leq 50$                 |                      | $2y_1 + y_2 \geq 50$                 |
|                      | $2x_1 + 2x_3 \leq 24$                |                      | $y_1 \geq 0, y_2 \geq 0, y_3 \geq 0$ |
|                      | $x_1 \geq 0, x_2 \geq 0$             |                      |                                      |
| (c) <b>Maximize:</b> | $z = 8x_1 + 10x_2 + 7x_3$            | (d) <b>Minimize:</b> | $w = 6y_1 + 10y_2$                   |
| <b>subject to:</b>   | $x_1 + 3x_2 + 2x_3 \leq 10$          | <b>subject to:</b>   | $3y_1 + 5y_2 \geq 15$                |
|                      | $x_1 + 5x_2 + x_3 \leq 8$            |                      | $4y_1 + 7y_2 \geq 20$                |
|                      | $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ |                      | $y_1 \geq 0, y_2 \geq 0$             |

**Problem 11.** How long will it take the general level of prices in the economy to double at a 4% annual inflation rate? A 7% annual inflation rate?

**Problem 12.** Find the future value of the ordinary annuity.

- $R = 1250$ ; 5% interest compounded semiannually for 18 years.
- $R = 4600$ ; 8.73% interest compounded quarterly for 9 years.
- $R = 42,000$ ; 10.05% interest compounded semiannually for 12 years.

**Problem 13.** Find the future value of the annuity due.

- $R = 600$ ; 6% interest compounded annually for 8 years.
- $R = 25,000$ , 8% interest compounded quarterly for 6 years.
- $R = 8000$ , 4.5% interest compounded semi-annually for 10 years.

**Problem 14.** Find the present value of each ordinary annuity.

- Payments of \$1400 each year for 8 years at 7% compounded annually.
- Payments of \$50,000 quarterly for 10 years at 4% compounded quarterly.
- Payments of \$18,579 every 6 months for 8 years at 5.4% compounded semiannually.

**Problem 15.** Find the truth table for each compound statement.

- |                           |  |                                       |
|---------------------------|--|---------------------------------------|
| (a) $p \wedge (q \vee r)$ | (b) $\sim p \vee (q \rightarrow \sim r)$ | (c) $p \wedge \sim (r \rightarrow q)$ |
|---------------------------|--|---------------------------------------|

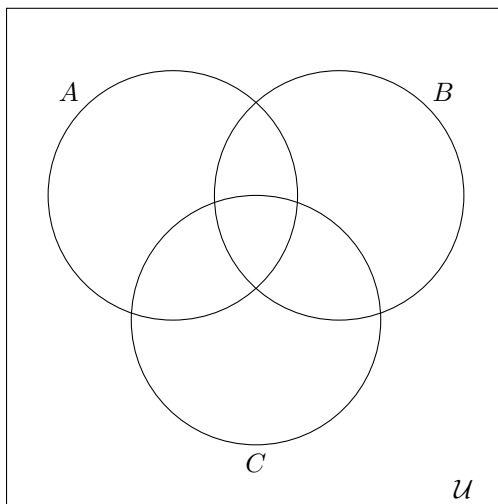
**Problem 16.** Solve the following problems.

- If  $n(A) = 5$ ,  $n(B) = 12$ , and  $n(A \cap B) = 4$ , what is  $n(A \cup B)$ ?
- If  $n(A) = 15$ ,  $n(B) = 30$ ,  $n(A \cup B) = 33$ , what is  $n(A \cap B)$ ?
- If  $n(B) = 9$ ,  $n(A \cap B) = 5$ , and  $n(A \cup B) = 22$ , what is  $n(A)$ ?
- If  $n(A \cap B) = 5$ ,  $n(A \cup B) = 38$ , and  $n(A) = 13$ , what is  $n(B)$ ?

**Problem 17.** Country-western songs often emphasize three basic themes: love, prison, and trucks. A survey of the local country-western radio station produced the following data:

- 12 songs were about a truckdriver who was in love while in prison;
- 13 were about a prisoner in love;
- 28 were about a person in love;
- 18 were about a truckdriver in love;
- 33 were about people not in prison;
- 18 were about prisoners;
- 15 were about truckdrivers who were in prison;
- 16 were about truckdrivers who were not in prison.

- (a) Use the information above to identify the values of each region in the following Venn diagram if we let  $A$  represent the set of people who are in love,  $B$  represent the set of people who are in prison, and  $C$  represent the set of people who drive trucks.



- (b) How many songs were surveyed?  
(c) How many songs were about truckdrivers?  
(d) How many songs were about prisoners who are in love but are not truckdrivers?  
(e) How many songs were about people not in love?

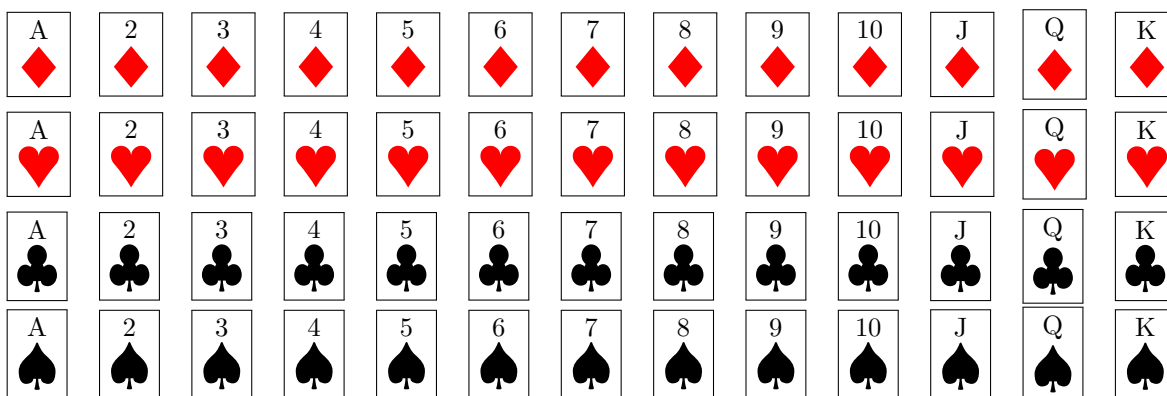
**Problem 18.** Five hundred and fifty college seniors attending a career fair at a university were categorized according to gender and according to primary career motivation, as summarized in the following table.

	Primary Career Motivation			Total
	Money	Allowed to Be Creative	Sense of Giving to Society	
Male	82	123	101	306
Female	68	93	83	244
Total	150	216	184	550

If one of these students is to be selected at random, find the probability that the student selected will satisfy each condition.

- The student is female.
- The student is motivated primarily by a sense of giving to society, given that the student is male.
- The student is not motivated primarily by creativity.
- The student is not motivated by money or a sense of giving back to society, given that the student is female.

**Problem 19.** Consider the layout of a standard deck of 52 cards.



Draw a single card. Find the odds in favor of drawing a

- King or Queen
- Face card
- Prime number
- Spade and an Ace

**Problem 20.** For mutually exclusive events  $R_1$ ,  $R_2$ , and  $R_3$ , we have that  $P(R_1) = 0.15$ ,  $P(R_2) = 0.55$ ,  $P(R_3) = 0.30$ . Also,  $P(Q | R_1) = 0.40$ ,  $P(Q | R_2) = 0.20$ , and  $P(Q | R_3) = 0.70$ . Determine the following probabilities.

- $P(R_1 | Q)$
- $P(R_2 | Q)$
- $P(R_3 | Q)$

**Problem 21.** How many distinguishable arrangements of letters are possible in the following words?

- IDIOSYNCRACIES
- INDIANA
- ISOMORPHISM
- DETARTRATED

**Problem 22.** In a club with 9 male and 11 female members, a 5 member committee will be randomly chosen. Find the probability that the committee contains the following. Round your answers to four decimal places.

- (a) All men.
- (b) 3 men and 2 women.
- (c) 2 men and 3 women.
- (d) At least 4 women.
- (e) No more than 2 men.

**Problem 23.** Toss a coin 5 times and the number of tails that come up is tallied. What is the probability that you toss

- (a) Exactly 3 tails.
- (b) At least 2 tails.
- (c) At most 4 tails.

**Problem 24.** Find the mean for each set of data.

- (a) 67, 89, 75, 86, 100, 93
- (b) 15.3, 27.2, 14.8, 16.5, 31.8, 40.1, 18.9, 28.4, 26.3, 35.3

(c)

Value	Frequency
4	6
6	1
9	3
15	2

(d)

Interval	Frequency
40–44	3
45–49	6
50–54	7
55–59	14
60–64	3
65–69	2

**Problem 25.** Find the range and standard deviation for each set of numbers.

- (a) 72, 61, 57, 83, 52, 66, 85
- (b) 51, 58, 62, 64, 67, 71, 74, 78, 82, 93
- (c) 17, 57, 48, 13, 26, 3, 36, 28, 21, 9, 40