Convergence Test Summary

Math 208, Professor Maltenfort, instructor

$\sum(a_i + b_i) = \sum a_i + \sum b_i$, and $\sum c\,a_i = c \sum a_i$.

Geometric Series: $a + ar + ar^2 + \cdots = \frac{a}{1-r}$ if $|r| < 1$ (diverges $|r| \geq 1$)

$\mathbf{i^{th}}$ Term Test for Divergence: If $\lim_{i\to\infty} a_i \neq 0$ then $\sum a_i$ diverges.

(Direct) Comparison Test: Suppose $|a_i| \leq b_i$.

- If $\sum b_i$ converges, then $\sum a_i$ converges.
- If $\sum a_i$ diverges, then $\sum b_i$ diverges.

Ratio Comparison Test: For $a_i \geq 0$ and $b_i \geq 0$, let $L = \lim_{i\to\infty} \dfrac{a_i}{b_i}$

- If $L = 0$ and $\sum b_i$ converges, then $\sum a_i$ converges.
- If $L = \infty$ and $\sum b_i$ diverges, then $\sum a_i$ diverges.
- If $0 < L < \infty$, $\sum a_i$ converges if and only if $\sum b_i$ converges.

Alternating Series Test: An alternating series converges provided the terms have absolute values which are decreasing and go to zero.

Integral Test: If $f(x)$ is positive and decreasing, and $a_i = f(i)$, then $\int_1^\infty f(x)\,dx$ converges if and only if $\sum_{i=1}^\infty a_i$ converges.

p-Series Test: The series $\sum \dfrac{1}{i^p}$ converges if $p > 1$ and diverges if $p \leq 1$. The special case $p = 1$ is the **Harmonic Series**.

Ratio and Root Tests: Let $L = \lim\limits_{i\to\infty} \left|\dfrac{a_i}{a_{i-1}}\right|$ or $L = \lim\limits_{i\to\infty} \sqrt[i]{|a_i|}$.

- If $L < 1$, then $\sum a_i$ converges absolutely.
- If $L > 1$ or $L = \infty$, then $\sum a_i$ diverges.
- *Optional:* If the a_i are coefficients in a power series $\sum a_i(x - x_0)^i$, then the radius of convergence is $R = 1/L$. That is, the series converges absolutely for $|x - x_0| < R$, diverges for $|x - x_0| > R$, and anything can happen when $|x - x_0| = R$.

$\sum a_i$ **converges absolutely** if $\sum a_i$ converges and $\sum |a_i|$ converges.
$\sum a_i$ **converges conditionally** if $\sum a_i$ converges, but $\sum |a_i|$ diverges.
(Note: If $\sum |a_i|$ converges, then automatically $\sum a_i$ converges.)