Conic Sections

Circle

The circle with centre $(0, 0)$ and radius $r$ has the equation:

$$x^2 + y^2 = r^2$$

The circle with centre $(h, k)$ and radius $r$ has the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

General Form of the Circle

An equation which can be written in the following form (with constants $D, E, F$) represents a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Formal Definition

A circle is the locus of points that are equidistant from a fixed point (the center).

Conic Section

If we slice one of the cones with a plane at right angles to the axis of the cone, the shape formed is a circle.
**Parabola**

**Parabola with Vertical Axis**

A parabola with focal distance $p$ has equation:

$$x^2 = 4py$$

If the axis of a parabola is **vertical**, and the vertex is at $(h, k)$, we have

$$(x - h)^2 = 4p(y - k)$$

**Parabola with Horizontal Axis**

In this case, we have the relation:

$$y^2 = 4px$$

If the axis of a parabola is horizontal, and the vertex is at $(h, k)$, the equation becomes

$$(y - k)^2 = 4p(x - h)$$

**Formal Definition**

A parabola is the locus of points that are equidistant from a point (the focus) and a line (the directrix).

**Conic Section**

If we slice a cone parallel to the slant edge of the cone, the resulting shape is a parabola.
Ellipse

**Horizontal Major Axis**

The equation for an ellipse with a horizontal major axis and center (0,0) is given by:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

The foci (plural of ‘focus’) of the ellipse (with horizontal major axis) are at \((-c,0)\) and \((c,0)\) where \(c\) is given by:

\[
c = \sqrt{a^2 - b^2}
\]

The vertices of an ellipse are at \((-a,0)\) and \((a,0)\).

A parabola with horizontal major axis and with center at \((h, k)\) is given by:

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]

If the major axis is *vertical*, then the formula becomes:

\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
\]

We always choose our \(a\) and \(b\) such that \(a > b\).

**Vertical Major Axis**

**Formal Definition**

An ellipse is the locus of points whereby the sum of the distances from 2 fixed points (the foci) is constant.

**Conic Section**

When we slice one of the cones at an angle to the sides of the cone, we get an ellipse, as seen in the view from the top (at right).
**Hyperbola**

**North-south Opening**

For a north-south opening hyperbola:

\[
\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1
\]

The slopes of the asymptotes are given by:

\[\pm \frac{a}{b}\]

For a "north-south" opening hyperbola with centre \((h, k)\), we have:

\[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1
\]

**East-west Opening**

For an east-west opening hyperbola:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

The slopes of the asymptotes are given by:

\[\pm \frac{b}{a}\]

For an "east-west" opening hyperbola with centre \((h, k)\), we have:

\[
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
\]

**Formal Definition**

A hyperbola is the locus of points where the difference in the distance to two fixed foci is constant.
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